System Parameter Identification for Uncertain Two Degree of Freedom Vibration System

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ABTRRACT

Generally there are discrepancies between the responses of the analytical model and those of its true model under various excitations and boundary conditions. The factors of these deviations can be categorized into two group; one is the uncertainty in the analytical modeling caused by the assumptions used in the development of the mathematical model, and the other is the environmental disturbances such as the Coulomb friction, inertia, measurement noise and so on. Obtaining exact analytical model is very crucial to characteristic analysis of the system and design improvement based on this analysis. Therefore, model improvement has been more important in vibration problems as design and modeling become refined and further intricate. In this project, the model improvement of analytical governing equation is studied for two-degree-of-freedom model using experimental results. Based on vibration test results, existing analytical system is identified providing updated system characteristics such as mass and stiffness distributions. In addition, the disturbance of this system is estimated by disturbance observer algorithm [3-5].

Key words: Model updating, Disturbance observer, System parameter identification, Two-degree-of-freedom vibration system, System uncertainty.

1. Introduction

This project aims to improve a simple analytical model which has two lumped masses and stiffness's, based on analysis of experimental results for the real system by obtaining updated system parameters and estimating disturbance of the system. Concretely, through this project, following objectives will be achieved.

a) Build 2-DOF theoretical model and find analytical solution.

b) Construct hardware to represent this proposed model.

c) Acquire skill in designing, conducting experiments and analyzing the results.

d) Develop algorithms to update system parameters and to extract the disturbance from system.

Figure 1 shows the theoretical model with two degree of freedom (so called 2-DOF) conceptually which has two lumped mass m_1 and m_2 and connected springs with stiffness k_1, k_2. The numerical model of this system expressed in Eq. 1 and subscript 'a' means analytical value of mass and stiffness.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = F + \hat{d} \text{ or } M_a\ddot{X} + K_aX = F + \hat{d} \qquad \text{Eq. 1}$$

Even though we try to make this theoretical model, there are lots of factors which make discrepancies between real model and theoretical model. These factors are expected like following; real springs themselves have not only stiffness but also damping. Also, there should be some energy dissipation caused by clearance in attachment between spring and mass. Friction between mass and contact surface cannot removed perfectly although we implement very smoothing bearing system in this region. Measured mass in numerical model also should be reconsidered. To assume masses are lumped point, spring forces or excitation forces should be applied exactly to the center of gravity point of the mass m. If there is misalignment between excitation point and center of gravity point, rotational motion may be happen affecting the linear motion of mass. Because these factors are not considered in the numerical model, Eq. 1 cannot describe the motion of m_1, m_2 exactly. Therefore, model update is needed to describe the response as close to real model as possible.

Through this project, we update the equation of theoretical model, Eq. 1 into model Eq. 2. The subscript of 'r' for M and K and 'd' in force term means 'real' and 'disturbance' respectively.

$$M_r\ddot{X} + K_rX = F + d \qquad \text{Eq. 2}$$

Based on test results, M and K are updated and the disturbances, d remaining in the updated system is assessed using disturbance observer method. Using the updated parameters (M_u, K_u), the disturbance observer method derive the estimated disturbance $\hat{d}$ to obtain a model that is closer to the real system (M_r, K_r, d).

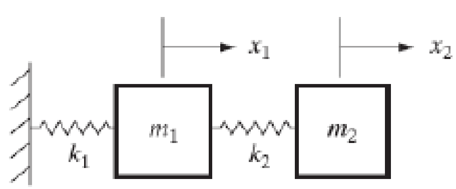


Figure 1. Schematic of two-degree-of-freedom model

2. Hardware Design and Experimental Setup

2.1 Hardware Design

For this research, the 2-DOF hardware was designed as shown in Figure 2. The hardware system consists of two masses connected in series by two springs with base frame and linear guide. Each mass and linear guide carriage are connected to reduce the friction. Also, linear guide prevent a moment that generated by excitation force that has not impact point on center of mass. Each mass consists of three component. Therefore this system able to change weight of mass by attach or detach mass component. Also, in order to change spring coefficient, it is able to change a spring.

2.2 Experimental Setup

To obtain experimental result, the hardware system is excited by shaker and impact hammer. Load cell is installed in shaker and impact hammer at impact point, so that it measure the interaction force between hardware systems. Two accelerometer sensors are attached to each mass as shown in the Figure 3. Each analog sensor data are regulated by the signal conditioner and those analog data are processed to digital data by the data acquisition system (so called DAQ).

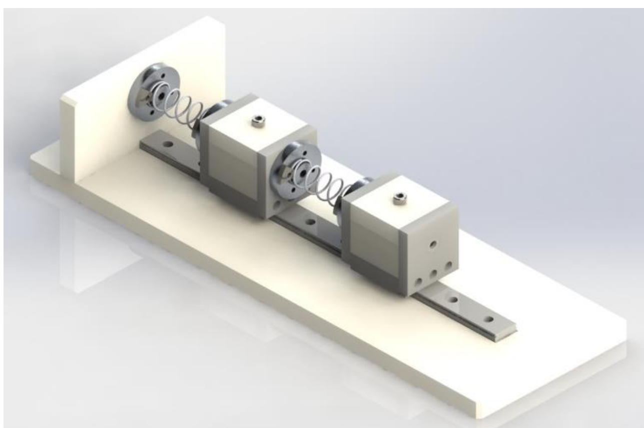


Figure 2. 3D CAD model of 2-DOF hardware system

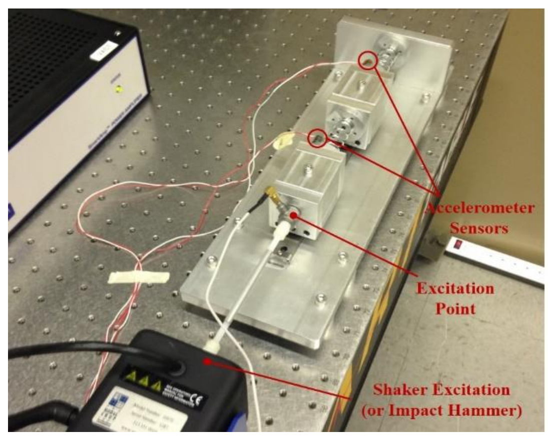


Figure 3. Experimental Setup

3. Algorithm and Experimental Result for the System Parameter Identification

3.1 Model Updating Algorithm

Figure 4 shows why mass update is needed using 1-DOF model. If spring force is applied to exact center of gravity point of body which has weight of m (Figure 4(a)), there exist only linear motion of m. Therefore, we can assume the mass as lumped mass and Eq. 3 can be used. In this theoretical case, linear kinetic energy is preserved; impact force, $\hat{F}$ cause initial velocity, v_0 with amount of $\hat{F}/m$ and the mass has kinetic energy corresponding. This energy is converted into spring potential energy and partly dissipated damping. However, if there is misaligning between these two points (Figure 4(b)), corresponding moment is induced. This moment cause extra energy to be dissipated in the contact region between body and contact surface. In this case, energy preservation can be expressed as Eq. 4. We can easily guess x_2 is smaller than x_1. If we want assume this real system is theoretical, mass should be rescaled into effective mass, m_e which makes $x_1 = x_2$. Rearrange Eq. 3, Eq. 4 on the condition that $x_1 = x_2$, we can get the Eq. 5 and guess effective mass will be smaller than measured mass.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{\hat{F}}{m}\right)^2 = \frac{1}{2}kx_2^2 + E_{damp} \quad \text{Eq. 3}$$

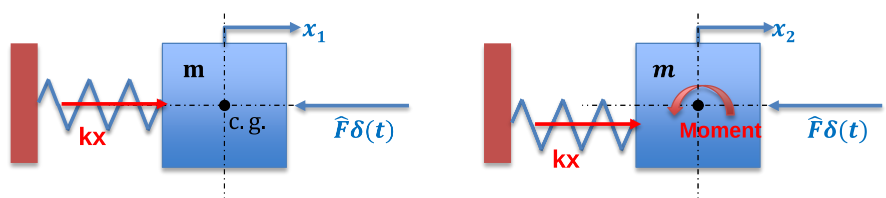
$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{\hat{F}}{m}\right)^2 = \frac{1}{2}kx_2^2 + E_{damp} + E_{Mloss} \quad \text{Eq. 4}$$

$$m_e = \frac{m}{1-2mE_{Mloss}/\hat{F}} < m \quad \text{Eq. 5}$$

To update measured masses into effective masses, response function of analytical model is optimized to match well the response function measured in the experiments. 1-DOF mass and corresponding spring system is constructed and excited by the impact hammer and their frequency response for acceleration is measured. This response function is converted into response function for displacement by dividing ω^2, which is designated as $H(\omega)$. Analytical frequency response function is expressed as Eq. 6.

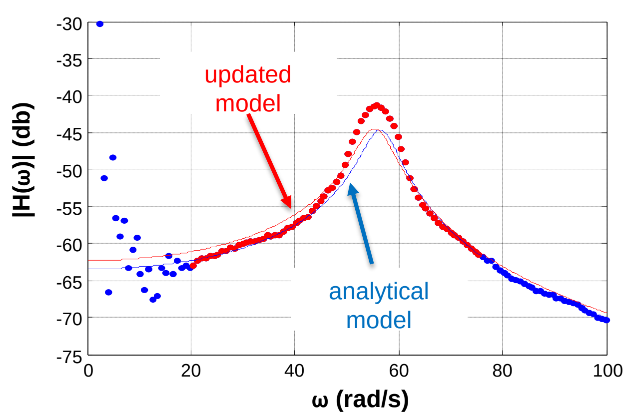
$$|H(j\omega)| = \frac{1}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}} \quad \text{Eq. 6}$$

Using this function and test results, optimization problem is described, which is shown in Figure 5. Damping coefficients of Eq. 6 is determine by 3dB rules using test results. Figure 5 shows the curve fitting results of transfer functions before and after updating. Optimization results in decrease by 9% in mass 1 and slightly increase in mass 2. Detail values can be checked in Table 1.



(a) Theoretical model (b) Real model

Figure 4. Effect of spring misaligning to center of gravity of mass



- min F : Objective Function

  F= $\left(|H(j\omega)|_{test} - |H(j\omega)|_{opti}\right)^2$
- variable : m
- constraint : $\omega_n|_{test} = \sqrt{k/m}$
- Results : F : 215.98→ 167.89

Figure 5. Mass update effect on transfer function and set up for optimization problem to find m_e

K matrix is updated based on 2-DOF test results. Figure 6 shows the test system conceptually. Impact force is applied to m_2 and accelerations of m_1 and m_2 are measured. As a test results, we obtained the two frequency transfer functions; one is transfer function between response of m_1 and force input to $m_2(H_{21}(\omega))$ and the other is between m_2 and force input to m_2 $(H_{22}(\omega))$. K matrix can be founded by the Eq. 7 which composed of eigenvector matrix P, diagonal matrix of natural frequencies which are determined from test results.

$$K = M^{1/2} P diag(\omega_i^2) P M^{\frac{1}{2}} \qquad \text{Eq. 7}$$

Figure 7 shows the frequency response function, $H_{21}(\omega)$, from test results for this system. As we can guess, two clear natural frequencies are shown in the chart. To determine the natural frequency, ω, modal damping coefficient, ζ, and peak values of response function, peak regions are fitted with 3rd polynomials. ith modal damping coefficient is calculated with ith natural frequency, $\omega_i$and ω_b, ω_c whose meaning is shown in Figure 7. ith mode shape, u_i is determined using Eq.9 and 10. With these information, finally K matrix is determined and updated values are shown in Table 1. Two tests repeated and corresponding two $[k_1, k_2]$ were determined and averaged. Generally, updated stiffnesses decrease by about 10% and k_1 decrease more than k_2.

$$\zeta_i = \frac{(\omega_b - \omega_a)}{\omega_i} \qquad \text{Eq. 8}$$

$$u_i = [a_1 \; a_2]^T \qquad \text{Eq. 9}$$

$$\begin{bmatrix} a_1^2 & |a_1 a_2| \\ |a_2 a_1| & a_2^2 \end{bmatrix} = \begin{bmatrix} |2\zeta_i \omega_i^2||H_{11}(\omega_i)| & |2\zeta_i \omega_i^2||H_{21}(\omega_i)| \\ |2\zeta_i \omega_i^2||H_{12}(\omega_i)| & |2\zeta_i \omega_i^2||H_{22}(\omega_i)| \end{bmatrix} \qquad \text{Eq. 10}$$

Finally updated effect is validated comparing numerical transfer functions before and after update with test results which are shown in Figure 8. The chart indicates there is no considerable improvement at mode 1; the agreement of two numerical model with test results is almost same. However, at mode 2, updating model is better matched to the test results; deviation of natural frequency with test results is reduced from 2.5% to 0.4% and from -9.8% to 0.3% for mode shape.

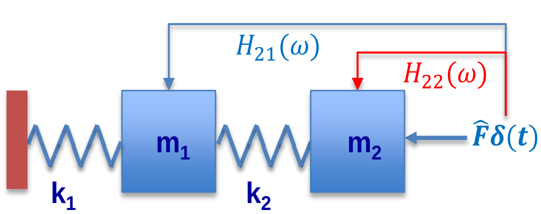


Figure 6. 2-DOF test system

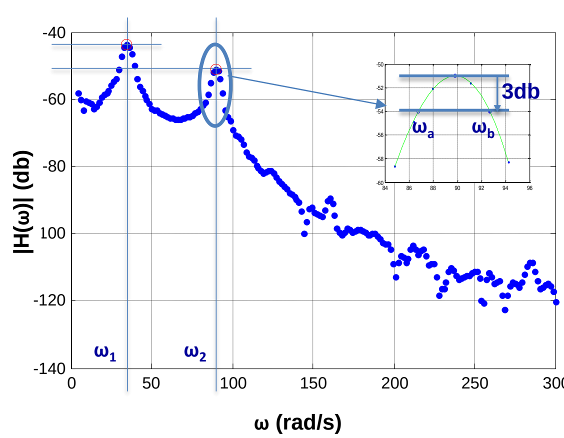


Figure 7. Test results: frequency response function for m_1

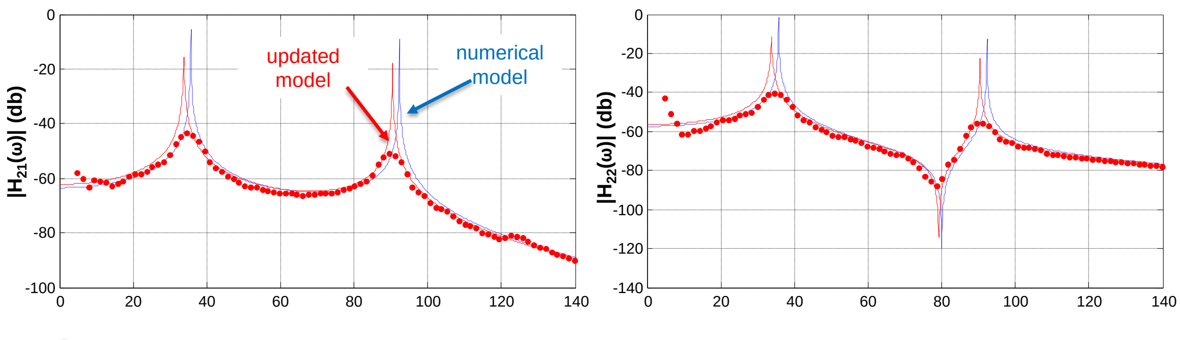


Figure 8. Numerical transfer function before and after update with test results

Table 1. System update results and effects on modal characteristics

	position	mass	stiffness		mode	ω_i	ζ_i	u(a1/a2)
numerical	1	0.465	1489.1	numerical	mode1	35.673	0.010	0.624
	2	0.440	1489.1		mode2	92.285	0.010	-1.516
updated	1	0.423	1297.3	updated	mode1	33.784	0.088	0.625
	2	0.446	1357.3		mode2	90.431	0.032	-1.687
				test	mode1	34.733	0.088	0.679
					mode2	90.071	0.032	-1.681
% deviation w.r.t. numerical model				% deviation w.r.t. test result				
updated	1	-9.0%	-12.9%	updated	mode1	-2.7%		-8.0%
	2	1.4%	-8.8%		mode2	0.4%		0.3%
				numerical	mode1	2.7%		-8.1%
					mode2	**2.5%**		**-9.8%**

3.2 Disturbance Observer

Mechanical Systems interact with environments, and thus they cannot be free from environmental disturbances. Most of the mechanical systems exhibit nonlinear behavior due to the disturbances such as Coulomb friction, inertia, sensor noise and so on. The disturbance observer estimates the disturbance from the input and measurement output. Figure 9 shows a block diagram of the disturbance observer method. $G_R(s)$ is a transfer function of actual plant dynamics model. $G_A(s)$ is a transfer function of mathematical analytical model (called nominal model). A disturbance, d, is exerted to the plant so that the output y is affected by the disturbance. The actual output is

$$y = G_R(s)[u + d] \qquad \text{Eq. 11}$$

By inverse transfer function of analytical model, the estimated input can be described as

$$\hat{u} = G_A^{-1}(s)y \qquad \text{Eq.12}$$

Subtracting the input, u, from the estimated input, $\hat{u}$, the effect of the disturbance and the model discrepancy, $\hat{d}$, can be estimated as

$$\hat{d} = Q(s)[G_A^{-1}(s)y - u] \quad \text{Eq.13}$$

where $Q(s)$ is a filter to make realizable. In practice, the inverse transfer function, $G_A^{-1}(s)$, is not realizable by itself. However, $Q(s)G_A^{-1}(s)$ can be made realizable by letting the relative order of $Q(s)$ be equal or greater than that of $G_A(s)$. $Q(s)$, which satisfy above stated properties, has been suggested by Umeno and Hori to be [3]

$$Q(s) = \frac{1+\sum_{k=1}^{N-r} a_k(\tau s)^k}{1+\sum_{k=1}^{N} a_k(\tau s)^k} \quad \text{Eq.14}$$

where r must be equal or greater than the relative order of the transfer function of the nominal model. N is order of $Q(s)$ and $1/\tau$ is cut-off frequency of $Q(s)$. The coefficients a_k are usually chosen as the coefficients of a Butterworth. As shown in the block diagram of disturbance observer, if $G_A(s)$ is closer to $G_R(s)$, then it is able to get more exact estimated disturbance from the algorithm. Therefore, updated parameters are used to design the nominal transfer function $G_A(s)$.

For the verification, the proposed disturbance observer method is implemented in the 2-DOF hardware system. The hardware system is excited by shaker with 16 Hz. In order to check the disturbance observer algorithm, the low frequency disturbance is applied intentionally by human force while the shaker excite the system. This human disturbance measured by load cell as shown in Figure 10. By disturbance observer method, estimated disturbance graph is obtained as shown in Figure 11. To verify disturbance observer method, estimated human disturbance is separated from the estimated disturbance. Low pass filter is used to separate estimated human disturbance because that the measured human disturbance has low frequency. Figure 12 shows comparison with estimated and measured human disturbance. In this experiment, N, r and τ for $Q(s)$ are selected as 5, 4 and 0.002, respectively. Coefficients of a_k are chosen from 5th order Butterworth coefficients as a_1 =3.236068, a_2 =5.236068, a_3 =5.236068, a_4 =3.236068 and a_5 =1, respectively. As expressed earlier, updated parameters and transfer function $H_{22}(s)$ are used for this algorithm.

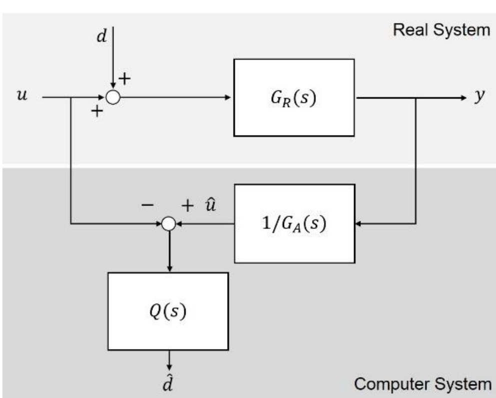


Figure 9. Block diagram of DOB

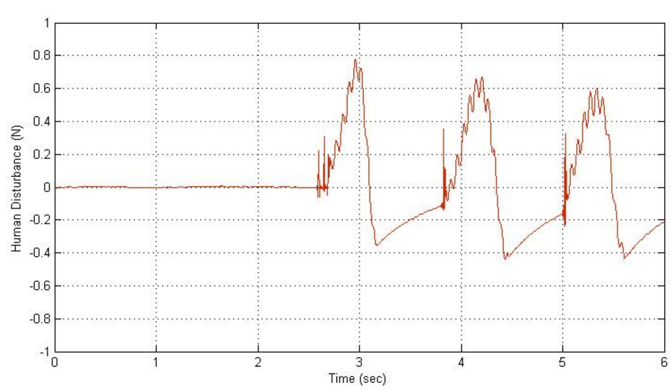


Figure 10. Measured human disturbance

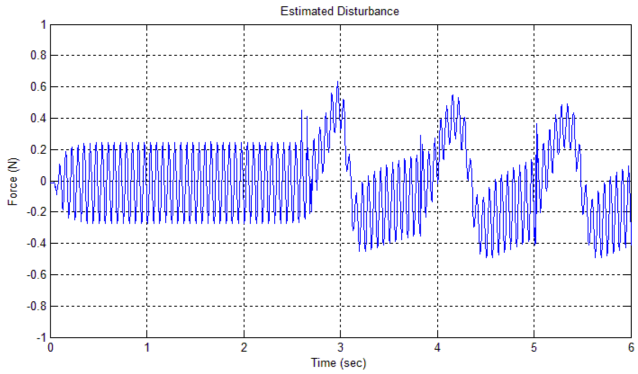


Figure 11. Estimated disturbance

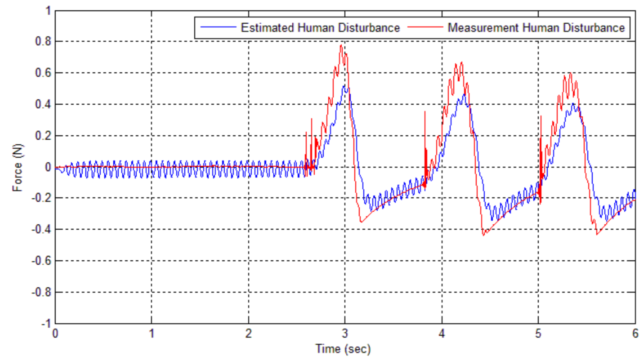


Figure 12. Estimated and measured human disturbance

4. Summary

On the basis of test results, two degree of freedom model (mass-spring) system is updated. Mass matrix update is conducted by optimizing the numerical transfer function to be well matched to the test results of 1-DOF system. Stiffness matrix was updated using modal characteristics from test result of 2-DOF system. Transfer function of updated model decrease the discrepancy with test results, especially at second mode. Using updated parameters, the nominal transfer function is designed and the disturbance observer method derive the estimated disturbance. To verify the method, intentional disturbance by human is separated from estimated disturbance with low pass filter. The estimated human disturbance agree well with the measured human disturbance.

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